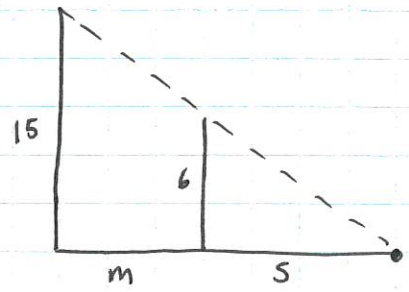
Related Rates #4

1)



$\frac{s}{6} = \frac{m+s}{15}$

MOMENT

$m = 40$

RATES

$\frac{dm}{dt} = 5 \text{ ft/sec}$

$15s = 6m + 6s$

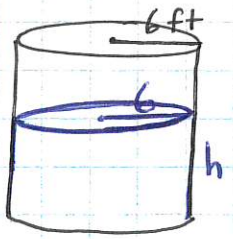
$9s = 6m$

$9\frac{ds}{dt} = 6\frac{dm}{dt}$

$\frac{ds}{dt} = \frac{6(5)}{9} = \frac{30}{9} = \frac{10}{3} \text{ ft/sec}$

$\frac{dm}{dt} + \frac{ds}{dt} = \frac{25}{3} \text{ ft/sec}$

2)



$V = \pi r^2 h$

* $r = 6$ ALWAYS

$V = 36\pi h$

MOMENT

RATES

$\frac{dV}{dt} = 8 \text{ ft}^3/\text{min}$

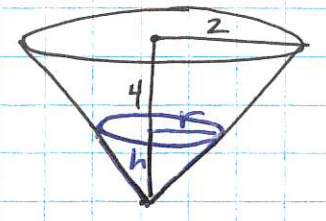
$\frac{dh}{dt} = ?$

$\frac{dV}{dt} = 36\pi \frac{dh}{dt}$

$8 = 36\pi \frac{dh}{dt}$

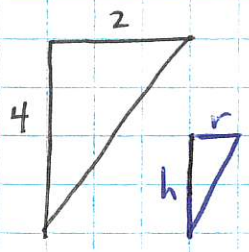
$\frac{dh}{dt} = \frac{2}{9\pi} \text{ ft/min}$

3)



$V = \frac{1}{3}\pi r^2 h$

*Get rid of r!



$r = \frac{1}{2}h$

$\frac{dr}{dt} = \frac{1}{2}\frac{dh}{dt}$

MOMENT

$r = 1$ in

$h = 2$ in

RATES

$\frac{dV}{dt} = -\frac{1}{2}$ in^3/sec

$\frac{dh}{dt} = ?$

$V = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 \cdot h$

$V = \frac{1}{12}\pi h^3$

$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \cdot \frac{dh}{dt}$

$-\frac{1}{2} = \frac{1}{4}\pi(4)\frac{dh}{dt}$

$-\frac{1}{2} = \pi\frac{dh}{dt}$

$\frac{dh}{dt} = -\frac{1}{2\pi}$ in/sec

OR

$V = \frac{1}{3}\pi r^2 h$

$\frac{dV}{dt} = \frac{1}{3}\pi r^2\frac{dh}{dt} + h\cdot\frac{2}{3}\pi r\frac{dr}{dt}$

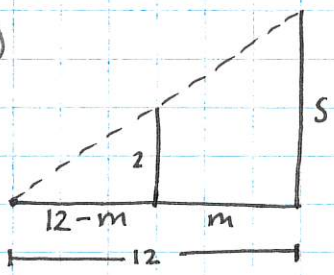
$-\frac{1}{2} = \frac{1}{3}\pi(1)\frac{dh}{dt} + 2\cdot\frac{2}{3}\pi(1)\cdot\frac{1}{2}\frac{dh}{dt}$

$-\frac{1}{2} = \frac{\pi}{3}\frac{dh}{dt} + \frac{2\pi}{3}\frac{dh}{dt}$

$-\frac{1}{2} = \pi\frac{dh}{dt}$

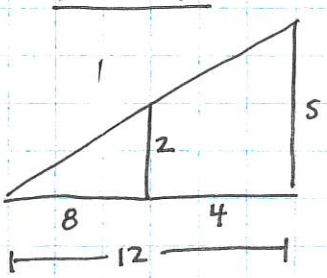
$\frac{dh}{dt} = -\frac{1}{2\pi}$ in/sec

4)



$\frac{2}{12-m} = \frac{s}{12}$

MOMENT



$\frac{2}{8} = \frac{s}{12}$

$8s = 24$

$s = 3$

MOMENT

$m = 4$ m

$s = 3$m

RATES

$\frac{dm}{dt} = -1.6$ m/sec

$24 = 12s - ms$

$0 = 12\frac{ds}{dt} - m\frac{ds}{dt} - s\frac{dm}{dt}$

$0 = 12\frac{ds}{dt} - 4\frac{ds}{dt} - 3(-1.6)$

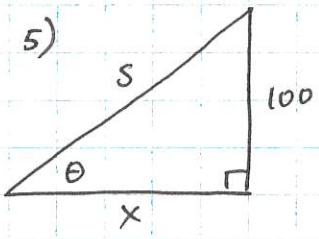
$0 = 8\frac{ds}{dt} + 4.8$

$-4.8 = 8\frac{ds}{dt}$

$-0.6 = \frac{ds}{dt}$

$\frac{ds}{dt} = -0.6$ m/sec

5)



MOMENT

$s = 200$

Rates

$\frac{dx}{dt} = 8$ ft/sec

$\tan\theta = \frac{100}{x}$

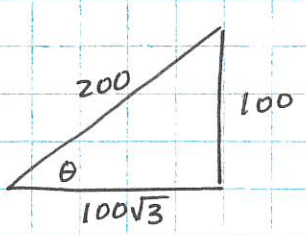
$\tan\theta = 100x^{-1}$

$\sec^2\theta \frac{d\theta}{dt} = -100x^{-2}\frac{dx}{dt}$

$\left(\frac{200}{100\sqrt{3}}\right)^2 \frac{d\theta}{dt} = \frac{-100}{(100\sqrt{3})^2}(8)$

$\frac{4}{3}\frac{d\theta}{dt} = -\frac{2}{75}$

MOMENT

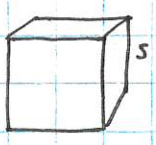


$\cos\theta = \frac{100\sqrt{3}}{200} = \frac{\sqrt{3}}{2}$

$\sec\theta = \frac{2}{\sqrt{3}}$

$\frac{d\theta}{dt} = -\frac{1}{50}$ rad/sec

6)



MOMENT

$s = 5$ ft

Rates

$\frac{ds}{dt} = -\frac{1}{10}$ ft/sec

$V = s^3$

$\frac{dV}{dt} = 3s^2\frac{ds}{dt}$

$\frac{dV}{dt} = -\frac{75}{10} = -\frac{15}{2}$ ft/sec

7)

MOMENT

$r = 5$ cm

Rates

$\frac{dr}{dt} = 0.01$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$

$\frac{dA}{dt} = 10\pi(0.01) = \frac{1}{10}\pi$ cm²/sec